Feedback Cooling of a Single Trapped Ion

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Based on a real-time measurement of the motion of a single ion in a Paul trap, we demonstrate its electromechanical cooling below the Doppler limit by homodyne feedback control (cold damping). The feedback cooling results are well described by a model based on a quantum mechanical master equation.

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Quantum optics, and more recently mesoscopic condensed matter physics, have taken a leading role in realizing individual quantum systems, which can be monitored continuously in quantum limited measurements, and at the same time can be controlled by external fields on time scales fast in comparison with the system evolution. Examples include cold trapped ions and atoms [1], cavity QED [2–5], and nanomechanical systems [6]. This setting opens the possibility of manipulating individual quantum systems by feedback, a problem which is not only of a fundamental interest in quantum mechanics, but also promises a new route to generating interesting quantum states in the laboratory. During the last decade a *quantum feedback theory* [7,8] has been developed, its basic ingredients are the interplay between quantum dynamics and the backaction of the measurement on the system evolution. First experimental efforts to realize feedback on various quantum systems [5,9–12] have been reported only recently. In this Letter we present an experiment on feedback control, i.e., feedback cooling, of a single trapped ion in front of a mirror. We establish a continuous measurement of the position of the laser driven ion by monitoring its fluorescence, then feed back a damping force proportional to the momentum thus demonstrating "cold damping" [13,14]. We use a quantum control theory based on a quantum optical modelling of the system dynamics and continuous measurement theory of photodetection. This provides us with a quantitative understanding of the experimental results.

We study a single $^{138}Ba^+$ ion in a miniature Paul trap which is continuously laser excited and laser cooled to the Doppler limit on its $S_{1/2}$ to $P_{1/2}$ transition at 493 nm, as outlined in Fig. 1. The ion is driven by a laser near the atomic resonance, and the scattered light is emitted into the radiation modes reflected by the mirror, as well as the other (background) modes of the quantized light field [15]. Light scattered into the mirror modes can either reach the photodetector directly, or after reflection from the mirror. This leads to an interference of the emitted photons, determined by the instantaneous position of the ion with respect to the mirror. Therefore, the motion of the ion (its projection onto the ion-mirror axis) modulates the photon counting signal, and is detected in the spectrum as a vibrational sideband [16], superimposed on the background shot noise. Of the three sidebands at about (1, 1.2, 2.3) MHz, corresponding to the three axes of vibration, we observe the one at $\nu = 1$ MHz. It has a width of about 400 Hz due to the Doppler cooling rate.

Our goal is to continuously read the position of the ion, and feed back a damping force proportional to the momentum to achieve feedback cooling. For a weakly driven atom the emitted light is dominantly elastic scattering at the laser frequency. The information on the motion of the ion is encoded in the sidebands of the scattered light, displaced by the trap frequency ν. For an ion trapped in the Lamb-Dicke regime, $\eta\sqrt{N} \ll 1$ (N is the mean excitation number

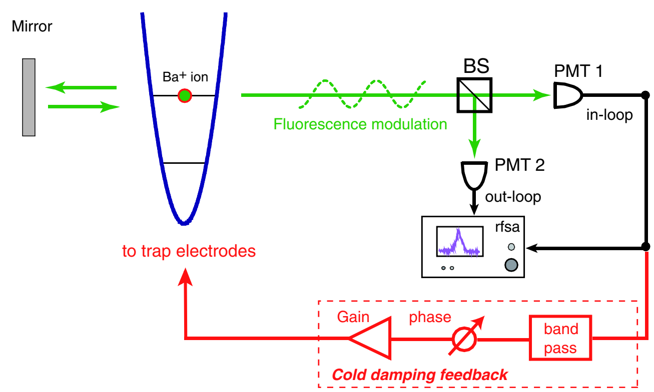

FIG. 1 (color online). A single $^{138}Ba^+$ ion in a Paul trap (parabola) is laser excited and cooled on its $S_{1/2}$ to $P_{1/2}$ transition at 493 nm. A retro-reflecting mirror 25 cm away from the trap and a lens (not shown) focus back the fluorescence onto the ion. The resulting interference fringes with up to 73% contrast are observed by a photomultiplier (PMT 1). The ion's oscillation in the trap creates an intensity modulation of the PMT signal which is observed as a sideband on a spectrum analyzer (rfsa) [16]. For feedback cooling, the sideband signal is filtered, phase-shifted, and applied to the ion as a voltage on a trap electrode.

of the motional oscillator), the motional sidebands are then suppressed by the Lamb-Dicke parameter, η, relative to the elastic component at the laser frequency. In our experiment, $\eta = 2\pi a_0/\lambda \sim 0.07$, a_0 being the rms size of the trap ground state. The light reaching the detector will thus be the sum of the elastic component and weak sidebands [16], a situation reminiscent of homodyne detection, where a strong oscillator beats with the signal field to provide a homodyne current at the detector. This physical picture allows us to formulate the continuous readout of the position of the ion as well as the quantum feedback cooling in the well-developed language of homodyne detection and quantum feedback.

The homodyne current at the photodetector (see Fig. 1) with the (large) signal from the elastic light scattering subtracted has the form

$$I_c(t) = \gamma\eta\langle\hat{z}\rangle_c(t) + \sqrt{\frac{\gamma}{2}}\xi(t). \qquad (1)$$

The first term is proportional to the conditioned expectation value of the position of the trapped ion, $\langle\hat{z}\rangle_c(t)$, and the second term is a shot-noise contribution with Gaussian white noise $\xi(t)$. We have defined $\hat{z} = a + a^\dagger \equiv z/a_0$ with a ($a^\dagger$) destruction (creation) operator of the harmonic oscillator, and we have assumed that the trap center is placed at a distance $L = n\lambda/2 + \lambda/8$ (n integer) from the mirror, corresponding to a point of maximum slope of the standing wave intensity of the mirror mode. The current $I_c(t)$ scales with $\gamma \propto \epsilon$, which is the light scattering rate into the solid angle $(4\pi\epsilon)$ of the mirror mode induced by the laser. The expectation value $\langle\cdot\rangle_c \equiv \mathrm{Tr}\{\cdot\rho_c(t)\}$ is defined with respect to a conditioned density operator $\rho_c(t)$, which reflects our knowledge of the motional state of the ion for the given history of the photocurrent. According to the theory of homodyne detection, $\rho_c(t)$ obeys the Ito stochastic differential equation [17]

$$d\rho_c(t) = -i\nu[a^\dagger a, \rho_c(t)]dt + \mathcal{L}_0\rho_c(t)dt + \sqrt{2\gamma\eta^2}\mathcal{H}\rho_c(t)dW(t), \qquad (2)$$

where $\mathcal{H}\rho_c(t) = [\hat{z}\rho_c(t) + \rho_c(t)\hat{z} - 2\langle\hat{z}\rangle_c(t)\rho_c(t)]$. The first line determines the unobserved evolution of the ion, including the harmonic motion in the trap with frequency ν and the dissipative dynamics, $\mathcal{L}_0$, due to photon scattering. The latter is given by

$$\mathcal{L}_0\rho = \Gamma(N+1)\mathcal{D}[a]\rho + \Gamma N\mathcal{D}[a^\dagger]\rho \qquad (3)$$

where we have defined the superoperator $\mathcal{D}[c]\rho \equiv c\rho c^\dagger - (c^\dagger c\rho + \rho c^\dagger c)/2$. The laser cooling rate $\Gamma \approx 400$ Hz and the steady-state occupation number $N = \langle a^\dagger a\rangle$ can either be estimated from the motional sidebands or deduced from the cooling laser parameters [1], which yields $N \approx 17$ for the present experimental parameters. The last term of Eq. (2) is proportional to the Wiener increment $dW(t) \equiv \xi(t)dt$ and corresponds to an update of the observers knowledge about the system according to a certain measurement result $I_c(t)$. In summary, Eq. (1) demonstrates that observation of the sidebands of the light scattered into the mirror mode provides us with information of the position of the ion, while the system density matrix is updated according to Eq. (2). This is the basis for describing feedback control of the ion, as shown in the following.

For feedback cooling, the vibrational sideband is extracted with a bandpass filter of bandwidth $B = 30$ kHz, shifted by $(-\pi/2)$, and amplified, and the resulting output voltage is applied to an electrode which is close to the trap inside the vacuum. Thereby we create a driving force which is proportional to (and opposed to) the instantaneous velocity of the ion and which thus adds to the damping of its vibration, i.e., cold damping. The overall gain of the feedback loop depends on the interference contrast, photomultiplier tube (PMT) characteristics, etc., It is varied electronically by setting the amplification G of the final amplifier in the loop. We can also set the phase to other values than the optimum of $(-\pi/2)$ to compare experiment and theory.

To analyze the result of the feedback we look at the changes in the sideband spectrum. The modified spectra require careful interpretation. The spectrum observed inside the feedback loop (“in-loop” or PMT 1 in Fig. 1) shows not only the motion of the ion, but the sum of the motion and the shot noise. As the feedback correlates these fluctuations, a reduction of the signal below the shot-noise level may occur, similar in appearance to signatures of squeezed light. This effect is known as “squashing” [18], or “anticorrelated state of light” in an opto-electronic loop [19], and it does not constitute a quantum mechanical squeezing of the fluctuations [20,21]. The effect on the motion can only be reliably detected by splitting the optical signal before it is measured and recording it outside the feedback loop with a second PMT (“out-loop” in Fig. 1), whose shot noise is not correlated with the motion.

In Fig. 2 we show spectra recorded with the spectrum analyzer, and measured outside and inside the feedback loop. The first curve of each row, showing the largest sideband, is the one without feedback (gain $G = 0$). The other two curves are recorded with the loop closed (gain values $G = 1.3$ and 8.7). The sub shot-noise fluctuations inside the loop, when the ion is driven to move in antiphase with the shot noise, are clearly visible in Fig. 2(f). The main cooling results are Figs. 2(b) and 2(c), which show the motional sideband reduced in size and broadened, indicating a reduced energy (proportional to the area under the curve) and a higher damping rate (the width). From case (b) to (c) the area increases, as the injected and amplified shot noise overcompensates the increased damping. As shown below, in our model the incorporation of quantum feedback competing with laser cooling predicts such behavior, i.e., the existence of an optimal gain for maximal cooling (for a detailed description cf. Ref. [22]).

We model the effect of the feedback force acting on the ion by extending Eq. (2) with the feedback contribution,

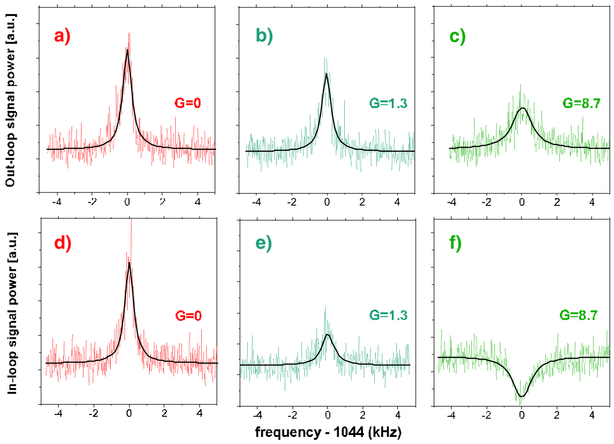

FIG. 2 (color online). Feedback cooling spectra. The vibrational sideband around $\nu = 1$ MHz is shown on top of the spectrally flat shot-noise background. The solid line represents a Lorentzian fitting function. The upper curves (a), (b), (c) are measured outside the feedback loop, while the lower curves (d), (e), (f) are the in-loop results. Spectra (a) and (d) are for laser cooling only, the other curves are recorded with feedback at the indicated gain values. The feedback phase is set to $(-\pi/2)$. The gain values indicate the settings of the final amplifier in the feedback loop.

$$[d\rho_c(t)]_{\rm fb} = -i\tilde{G} I_{{\rm fb},c}(t-\tau)[\hat{z}, \rho_c(t)], \tag{4}$$

where $I_{{\rm fb},c}(t)$ denotes the measured current after the feedback circuit. The latter is proportional to the voltage applied to the trap electrodes. All conversion factors between the feedback current and the actual force applied on the ion are included in the overall gain $\tilde{G} \propto G$. The time delay τ in the feedback loop preserves causality and is small compared to the fastest time scale ν^{-1} of the motion of the ion which allows us to consider the Markovian limit ($\tau \to 0^+$).

To obtain an expression for the feedback current $I_{{\rm fb},c}(t)$, we change into a frame rotating with the trap frequency ν and define the density operator, $\mu_c(t) \equiv \exp(i\nu a^\dagger a t) \times \rho_c(t) \exp(-i\nu a^\dagger a t)$, evolving on the (slow) cooling time scales. This is convenient due to the large separation between the time scale of the harmonic oscillations, ν^{-1} and the time scale of laser and feedback cooling in our experiment. For our experimental parameters, $\nu \gg B \gg \Gamma$, the feedback current for a phase shift of $(-\pi/2)$ has the form [22]

$$I_{{\rm fb},c}(t) = \left(\gamma\eta\langle\hat{p}\rangle_c(t) + \sqrt{\frac{\gamma}{2}}\Xi(t)\right)\cos(\nu t), \tag{5}$$

where $\langle\hat{p}\rangle_c(t) \equiv {\rm Tr}\{\hat{p}\mu_c(t)\}$, and $\hat{p} \equiv i(a^\dagger - a)$ is the momentum operator conjugated to $\hat{z}$. The first term in this current therefore provides damping for the motion of the ion. The second term of Eq. (5) describes the shot noise which passes through the electronic circuit and is fed back to the ion. The stochastic variable $\Xi(t)$ is Gaussian white noise on a time scale given by the inverse bandwidth B^{-1}, whereby $B \gg \Gamma$ implies that it is spectrally flat on the frequency range of the cooling dynamics.

For a full record of the photocurrent $I_c(t)$, Eqs. (2) and (4) determine the evolution of the ion's motional state in the presence of feedback. As it is impractical to keep track of the whole photocurrent in the experiment, we derive a master equation for the density operator averaged over all possible realizations of $I_c(t)$, $\mu(t)$. Along the lines of the Wiseman-Milburn theory of quantum feedback [7], for a phase shift of $(-\pi/2)$, we obtain the *quantum feedback master equation* [22]

$$\dot{\mu} = \mathcal{L}_0\mu - i\frac{\tilde{G}\gamma\eta}{4}[\hat{z}, \hat{p}\mu + \mu\hat{p}] - \frac{\tilde{G}^2\gamma}{16}[\hat{z}, [\hat{z}, \mu]]. \tag{6}$$

The second and third terms are the additional contributions due to the feedback. The part linear in $\tilde{G}$ induces damping of the motion of the ion, and the term quadratic in $\tilde{G}$ describes the effect of the fed back noise leading to diffusion of the momentum. The competition between laser cooling, damping, and injected noise leads to the characteristic behavior of the steady-state number expectation value

$$\langle n\rangle_{\rm ss} = \frac{N + \eta\gamma\tilde{G}(2N-1)/2\Gamma + \gamma\tilde{G}^2/8\Gamma}{1 + 2\eta\gamma\tilde{G}/\Gamma}. \tag{7}$$

For small gain, damping dominates, and the energy of the ion is decreased below the Doppler limit. For higher gain, the diffusive term describing the noise fed back into the system overcompensates cooling, i.e., heats the ion. Consequently, for $(-\pi/2)$ feedback phase and optimal gain conditions the steady-state energy is minimized. On the contrary, for a π phase shift $(-\hat{z})$ replaces $\hat{p}$ in the second term of Eq. (6), the feedback force then merely induces a frequency shift, $\Delta\omega = \tilde{G}\gamma\eta/2$, but no damping. Increasing the gain then always enhances the steady-state number expectation value, i.e., the mean ion energy. We now compare the theoretical predictions and the experimental results for the two selected feedback phases $(-\pi/2)$ and π.

The measured ion energy as a function of the feedback electronic gain is shown in Fig. 3. On the first side, as expected, for a $(-\pi/2)$ feedback phase, cooling by more than 30% below the Doppler limit is achieved, while further increase of the gain drives the shot noise and therefore heats the motion of the ion. On the other side, a π phase shift in the feedback loop does not yield any damping; in such conditions the motion of the ion is only driven. This results in an increase of the measured sideband area (as shown in the inset of Fig. 3), as well as a shift of the sideband center frequency (graph not shown). Both cases demonstrate good agreement between experiment and theory. Finally, let us stress that the optimal cooling rate is governed by the collection efficiency of the fluorescence going into the mirror mode, ϵ. In the experiments presented above, $\epsilon \approx 1\%$ leads to a decrease of the steady-state occupation number N from 17 to 12. For the experimen-

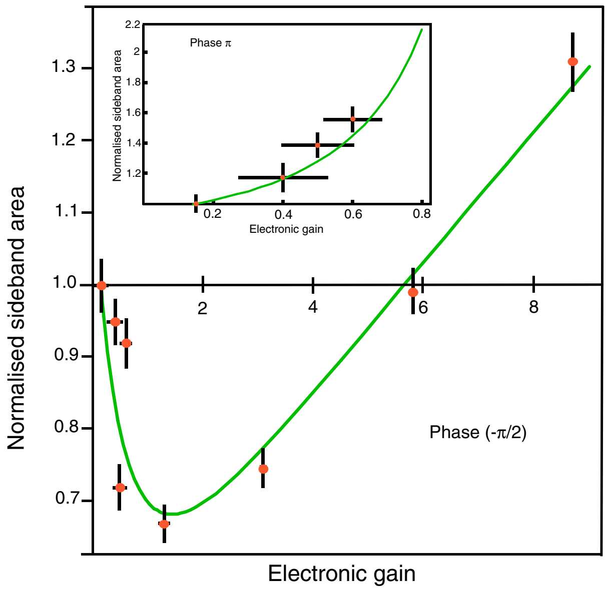

FIG. 3 (color online). Steady-state energy of the cooled oscillator: measured sideband area, normalized to the value without feedback, versus gain of the feedback loop, for $(-\pi/2)$ (main plot) and π (inset) feedback phase. The curves are the model calculations. The gain axis is scaled to the experimental values of the electronic gain.

tally realistic $\epsilon \approx 15\%$ and under optimal experimental cooling conditions a final state occupation $N \approx 3$ can be achieved, which would allow tests even further in the quantum regime.

To summarize, we have demonstrated real-time feedback cooling of the motion of a single trapped ion. Electromechanical backaction based on a sensitive real-time measurement of the motion of the ion in the trap allowed us to cool one motional degree of freedom by 30% below the Doppler limit. Unlike with laser cooling, the presented method allows us to cool one of the ion's motional mode without heating the two others, and our procedure can easily be extended to cooling all motional modes. The cooling process is shot-noise limited and the fraction of scattered photons recorded to observe the motion of the ion limits the ultimate cooling at optimum gain. The latter can yield a steady-state occupation number $N = 3$ for realistic experimental conditions, offering a possible way to efficiently cool the motion of ions unsuitable for sideband cooling.

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