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Geometric phase gate on an optical transition for ion trap quantum computation

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We propose a geometric phase gate of two ion qubits that are encoded in two levels linked by an optical dipole-forbidden transition. Compared to hyperfine geometric phase gates mediated by electric dipole transitions, the gate has many interesting properties, such as very low spontaneous emission rates, applicability to magnetic field insensitive states, and use of a co-propagating laser beam geometry. We estimate that current technology allows for infidelities of around 10^{-4}.

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One of the important and most difficult experimental efforts of quantum computation is the realization of almost perfect two-qubit gate operations. Currently it is believed that gate error probabilities of about 10^{-4} would be sufficiently low to allow for so-called efficient fault-tolerant quantum computing [1,2]. Strings of trapped ions are among the most promising candidates for the realization of a quantum computer. The lowest gate infidelity experimentally achieved with ion quantum gates is still around 3% [3]. The main limitations of this geometric phase gate come from spontaneous emission and magnetic field fluctuations [3,4].

Ion trap quantum computation can be implemented with two alternative qubit encodings: hyperfine ground state qubits and qubit states connected by optical transitions. For hyperfine qubits, the gate operations are performed by Raman coupling mediated by dipole transitions. Reference [3] used an encoding based on such a hyperfine transition. In such a setting, however, it is demanding to reduce spontaneous scattering below the required fault-tolerant level [5,6], because a tremendous amount of laser power is required. Recently, the use of Raman processes on quadrupole transitions was proposed for hyperfine qubits [7]. However, this strategy requires high laser powers to achieve short gate times.

Here, we propose a σ^z-type geometric phase gate on an optical transition to overcome some of the limitations present in the realization of [3]. For instance, the use of an optical quadrupole transition allows one to sufficiently reduce the likelihood of a spontaneous emission event. Also it is shown that magnetic field insensitive states can be used for the σ^z geometric gate on an optical transition. More interestingly, the gate can be executed with a co-propagating laser beam configuration, which reduces the errors from phase fluctuations between two laser beams [4]. With hyperfine qubits, on the contrary, only a counter-propagating scheme can be utilized for the gate. Finally, the proposed gate can be directly used as a logical two-qubit gate in dephasing-free subspaces because of the natural suitability of phase gates for such purposes [8].

The gate proposed here applies to any ion-qubit states connected by weak transitions, such as quadrupole transitions of Ca^+, Sr^+, and Ba^+. We first show that it is possible to realize a state-dependent displacement on the optical transitions with bichromatic laser radiation, and that for the most interesting detunings of the laser fields the coupling is maximized in a co-propagating geometry. In turn, the applicability of the gate to magnetic field insensitive states is explained. We then extend the scheme to two ions and carefully study the intrinsic complications leading to infidelities of the gate. We show that these can be compensated by spin-echo techniques, thus reducing the infidelities to a level of about 10^{-4}. We also discuss the connection between gate speed and the probability of spontaneous emissions during one gate operation, and show that the error due to it can also be reduced to a level of 10^{-4}. Finally, we briefly examine other more technically relevant errors arising from fluctuations of experimental parameters.

Consider a single two-level ion in a one-dimensional harmonic trap interacting with two laser beams detuned from the quadrupole transition of S to D. The Hamiltonian in the interaction picture and after the rotating wave approximation with respect to optical frequencies is given by

$$\hat{H}_I = \sum_{j=1,2} \hat{\sigma}^+ \left(\frac{\hbar\Omega_j}{2} e^{-i(\Delta_j t+\phi_j)} e^{i\eta_j(\hat{a}e^{-i\nu t}+\hat{a}^\dagger e^{i\nu t})} \right) + \text{H.c.}, \quad (1)$$

where $\hat{\sigma}^+ = |D\rangle\langle S|$, $\hat{a}$, and $\hat{a}^\dagger$ are the ladder operators of the oscillator, ν is the trap frequency, and Ω_j and η_j are the Rabi frequency and Lamb-Dicke parameter of the laser with detuning Δ_j and optical phase ϕ_j, respectively.

As will be detailed below, a state-dependent displacement operation appears by setting $\Delta_1 = \Delta$, $\Delta_2 = \Delta - \nu + \delta$ ($\delta \ll \nu$). In the Lamb-Dicke regime, at low laser intensity $\Omega_j \ll \Delta_j$, and ignoring terms faster than δ, a second-order perturbation yields the following effective interaction Hamiltonian:

$$\hat{H}_{\text{eff},1} = \left(\frac{\hbar\Omega_{\text{eff}}}{2} \hat{a} e^{-i\delta t} + \frac{\hbar\Omega^*_{\text{eff}}}{2} \hat{a}^\dagger e^{i\delta t} \right) \hat{\sigma}^z (+\text{LS}). \quad (2)$$

Here, $\Omega_{\text{eff}} = \left(\frac{\eta_1(\Delta-\nu+\delta/2)}{(\Delta-\nu)(\Delta-\nu+\delta)} - \frac{\eta_2(\Delta+\delta/2)}{\Delta(\Delta+\delta)}\right)\frac{\Omega_1\Omega_2}{2} e^{-i\phi_L}$, and LS denotes the light shifts coming from the transitions of carrier and the first motional sidebands and is given by $\sum_{j=1,2}\hbar\Omega_j^2\hat{\sigma}^z[-\frac{1}{4\Delta_j} - (\frac{\eta_j^2}{4(\Delta_j+\nu)} - \frac{\eta_j^2}{4(\Delta_j-\nu)})(\hat{n}+\frac{1}{2})]$, where $\hat{\sigma}^z = |D\rangle\langle D| - |S\rangle\langle S|$, $\hat{n} = \hat{a}^\dagger\hat{a}$, $\phi_L = \phi_1 - \phi_2 - \pi/2$. Neglecting LS for the moment, the effective Hamiltonian (2) describes the

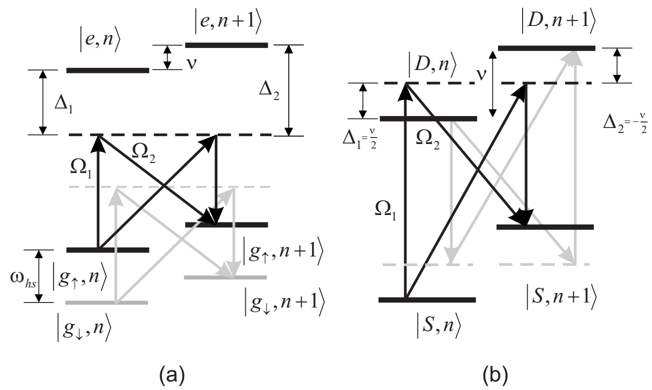

FIG. 1. Schematic representation of bichromatic laser frequencies to excite the motional states from $|n\rangle$ to $|n+1\rangle$ (a) for hyperfine qubit states connected via dipole transitions and (b) for states of the optical-transition qubit. To keep the figure simple, we omitted similar couplings connecting to $|n-1\rangle$ to $|n-2\rangle$, $|n+1\rangle$ to $|n+2\rangle$ and so on. On resonance, this ladder of interactions produces a displacement operation in the motional state space. In (a), $|g_{\downarrow,\uparrow}\rangle$ and $|e\rangle$ stand for two hyperfine ground states and an electric-dipole excited state, respectively.

desired state-dependent displacement operation [4]. The time evolution operator is found to be

$$\hat{U}(t) = \exp\left\{ -\frac{i}{\hbar}\left(\int_0^t dt' \hat{H}_{\rm eff}(t') - \frac{i}{2\hbar}\int_0^t dt' \int_0^{t'} dt'' [\hat{H}_{\rm eff}(t'),\hat{H}_{\rm eff}(t'')] + \cdots \right)\right\} \quad (3)$$

from the Magnus expansion related to the Baker-Campbell-Hausdorff formula $e^A e^B = \exp(A+B+\frac{1}{2}[A,B]+\cdots)$. With Eq. (3), the time evolution operator of Hamiltonian (2) can be obtained by $\hat{U}_1(t) = e^{(\alpha(t)\hat{a}^\dagger - \alpha(t)^*\hat{a})\hat{\sigma}_z} e^{i\Phi(t)\hat{\sigma}_z^2}$. Here, $\alpha(t) = \frac{\Omega_{\rm eff}}{2\delta}(1-e^{i\delta t})$ and $\Phi(t) = |\frac{\Omega_{\rm eff}}{2\delta}|^2(\delta t - \sin(\delta t))$. The ion moves periodically along a circular path in phase space of radius $|\hbar\Omega_{\rm eff}/2\delta|$ with periodicity $2\pi/\delta$, and the direction of motion is determined by the qubit state and ϕ_L. At $t=2\pi/\delta$, it returns to the original motional state and acquires the geometrical phase $\Phi_g = 2\pi|\Omega_{\rm eff}/2\delta|^2$. The latter phase depends only on the area enclosed by the trajectory, so both qubit states gain the same geometrical phase independently of ϕ_L.

We now show that, for optical-transition qubits, a co-propagating geometry maximizes the strength of the Raman coupling $\Omega_{\rm eff}$. For the case of hyperfine ground state qubits connected by dipole transitions, the detunings Δ_1, Δ_2 must be much larger than ν in Fig. 1(a), so $\Delta_1 \simeq \Delta_2 = \Delta$, which implies that $|\Omega_{\rm eff}| = |\frac{\eta_1 - \eta_2}{\Delta}| \frac{\Omega_1\Omega_2}{2}$. It is then essential, in order to achieve a nonvanishing coupling, to use a nonco-propagating laser beam configuration ($\eta_1 \neq \eta_2$). With quadrupole transitions, the detunings can be of the order of ν without considerable spontaneous emission. At the detunings $\Delta_1 = -\Delta_2 \approx \frac{\nu}{2}$, the coupling strength $|\Omega_{\rm eff}| = |-\frac{\eta_1}{\Delta_1} + \frac{\eta_2}{\Delta_2}| \frac{\Omega_1\Omega_2}{2}$ is maximized at $\eta_1 = \eta_2$. The Raman coupling with those detunings are depicted in Fig. 1(b). Most interestingly, the co-propagating geometry reduces optical phase fluctuations from path instabilities. Furthermore the co-propagation geometry also ensures that the displacement operation can be executed regardless of the ions' spacing. This is in contrast to a counterpropagating geometry where it is necessary to carefully control such spacings so as to have the proper laser phase on each ion [3].

Moreover, the symmetry of the detunings guarantees that the light shift in Eq. (2) disappears provided that both lasers' intensities coincide $\Omega_1 = \Omega_2$. Thus, it is not necessary to consider polarization states to equalize ac stark shifts of internal states from the two laser beams. Finally, state-dependent coupling is achieved without any restriction on the magnetic field properties of the states. The scheme proposed here is, therefore, applicable to magnetic field insensitive transitions, e.g., the quadrupole transitions of $^{43}Ca^+$ ion [9].

Now we extend the above consideration to two ions and study the two-qubit gate operation with the detunings $\Delta_1 = \frac{\nu}{2} - \frac{\delta}{2}$, $\Delta_2 = -\frac{\nu}{2} + \frac{\delta}{2}$ and the same Rabi frequency $\Omega_1 = \Omega_2 = \Omega$. We focus on the center of mass mode (CM). With two ions, the effective Hamiltonian is given by

$$\hat{H}_{\rm eff,2} = \frac{\hbar|\Omega_{\rm eff}|}{2}(\hat{a}e^{-i(\delta t+\phi_L)} + \hat{a}^\dagger e^{i(\delta t+\phi_L)})\hat{S}^z + \frac{4}{3}\frac{\hbar\eta|\Omega_{\rm eff}|}{2}(\hat{\sigma}_1^{-\Delta\varphi/2} \otimes \hat{\sigma}_2^{\Delta\varphi/2}) + \hat{O}(\eta^3), \quad (4)$$

where $|\Omega_{\rm eff}| = \frac{2\eta\Omega^2}{\nu}$, $\hat{S}^z = \hat{\sigma}_1^z + \hat{\sigma}_2^z$, $\hat{\sigma}_j^\varphi = e^{i\varphi}\hat{\sigma}_j^+ + e^{-i\phi}\hat{\sigma}_j^-$, and $\Delta\varphi = k_L \cos\theta(z_1 - z_2)$ is the laser phase difference between both ions, where k_L is the wave vector, θ is the angle between k_L and the trap axis, and z_j is the equilibrium position of the jth ion. The first term of Eq. (4) produces the evolution of a two-ion geometric phase gate: electronic states $|DD\rangle, |SS\rangle$ get both a $\Phi_g = 2\pi|\Omega_{\rm eff}/\delta|^2$ phase after $t=2\pi/\delta$, whereas states $|DS\rangle, |SD\rangle$ are not affected. At $\delta = \pm 2\Omega_{\rm eff}$, the obtained geometric phase Φ_g must be $\pi/2$ for the gate to be maximally entangling.

The second term in Eq. (4) is a Mølmer-Sørensen (MS) coupling [10], which does not occur for a geometric phase gate with hyperfine qubits. During the gate operation, the coupling transfers the population between $|DD\rangle$ and $|SS\rangle$, and between $|DS\rangle$ and $|SD\rangle$. Since populations do not change in an ideal σ^z geometric phase gate, the MS coupling increases the gate infidelity by producing population errors. We study the effect of MS coupling by a numerical integration of the exact Hamiltonian (1) of two ions. We sandwich the gate with two local pulses to turn it into a controlled-NOT gate and assess the performance of the gate through the state fidelity, the overlap between the final and the ideal states, using $|DD\rangle$ as the input state. All other input states display the same fidelities. As shown in the light-gray solid curve of Fig. 2(a), the infidelities are increased to 1% around $\eta = 0.06$ at the given simulation parameters mainly due to the MS coupling. By including MS coupling and keeping terms up to η^2 inside the exponent of Eq. (3), the time evolution operator for $t=2\pi/\delta$ and $\delta = 2\Omega_{\rm eff}$ can be approximated by

$$\hat{U}_2^{\rm gate} \approx e^{i(\pi/2)(\hat{S}^z/2)^2} e^{-i(2\pi\eta/3)(\hat{\sigma}_1^{-\Delta\varphi/2} \otimes \hat{\sigma}_2^{\Delta\varphi/2})}. \quad (5)$$

Here, we can see that the MS coupling changes the population of the final state at the end of the gate by $(\frac{2\pi\eta}{3})^2$, which

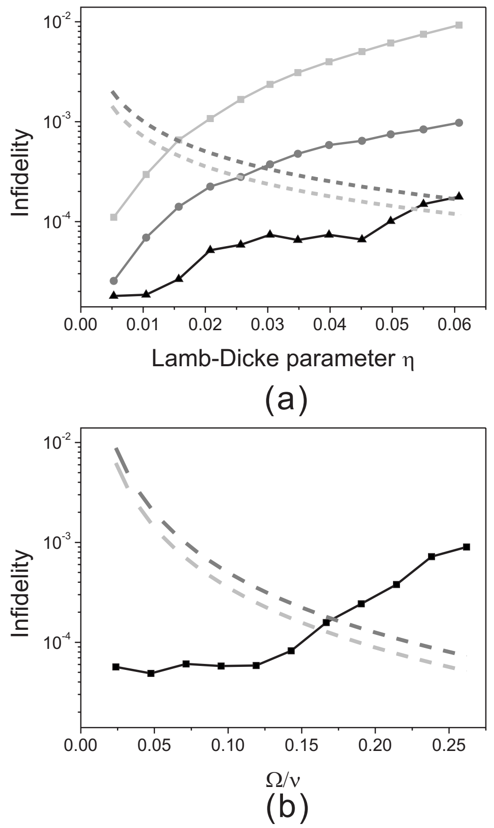

FIG. 2. (a) Gate error vs Lamb-Dicke fact η. The solid lines are the infidelities obtained from a numerical integration of the full Hamiltonian. The light gray line is obtained without spin-echo sequence, the gray one with spin-echo sequence and the black one with s pin-echo sequence and offset detruing $\delta_{\rm off}=(2\pi)20$ kHz. Here $\Omega=\nu/6$, $\nu=(2\pi)1.26$ MHz are used. The dominant error is caused by the MS coupling term. The dashed lines are infidelities caused by spontaneous decay of the metastable D state (in the case of Ca^+). The light gray curve is without spin-echo and gray curve includes a spin-echo sequence. The infidelities from spontaneous emissions are determined by the ratio between gate operation time and life time. (b) Gates error vs Rabi frequency. The black solid line is the infidelity as a function of Rabi frequency Ω with 10 μs pulse shaping, which is 10~100 times smaller than the gate time. The integration is executed with spin-echo sequence and $\delta_{\rm off}=(2\pi)20$ kHz at $\eta=0.056$ and $\nu=(2\pi)1.26$ MHz. Here the results are obtained by carefully controlling the phase difference between two lasers at the second pulse of the geometric phase gate to reduce an order of 0.1% errors. As in (a), dashed lines indicate errors caused by spontaneous emission.

is in agreement with light-gray solid curve of Fig. 2(a).

By using a careful spin-echo sequence and offset detuning $\delta_{\rm off}$, however, the MS coupling in the gate can be reduced. The spin-echo sequence exchanges the populations of the internal electronic states of both ions $[e^{i(\pi/2)(\hat{\sigma}_1^y+\hat{\sigma}_2^y)}]$ at the middle and end of the full gate operation. The $\hat{\sigma}^z$ gate is divided in two parts, and at each gate pulse the same electronic states get a $\pi/4$ phase after performing a closed circle phase space with the radius reduced by $\sqrt{2}$. Therefore, one needs to increase $\delta=2\sqrt{2}\Omega_{\rm eff}$ for the spin-echo sequence. Around the end of one gate pulse, the time evolution operator can be approximated by $\hat{U}_2^{\rm echo}=e^{i(\pi/4)(\hat{S}^z/2)^2}e^{-i(2\pi\eta/3\sqrt{2})(\hat{\sigma}_1^{-\Delta\varphi/2}\otimes\hat{\sigma}_2^{\Delta\varphi/2})}$. We have to add a $\pi/2$ phase to one ion $[e^{i(\pi/4)\hat{\sigma}_1^z}]$ in order to cancel out the MS operation. This can be shown by the following calculation:

$$e^{i(\pi/2)(\hat{S}^z/2)^2}=e^{i(\pi/2)(\hat{\sigma}_1^y+\hat{\sigma}_2^y)}\hat{U}_2^{\rm echo}e^{i(\pi/2)(\hat{\sigma}_1^y+\hat{\sigma}_2^y)}e^{i(\pi/4)\hat{\sigma}_1^z}\hat{U}_2^{\rm echo}. \tag{6}$$

For introducing a $\pi/2$ phase shift, one can either use a single off-resonant laser or change the distance between the ions. In Fig. 2(a), the gray solid curve shows that the spin-echo sequence indeed reduces the gate infidelity due to MS coupling.

Equations (5) and (6), however, are an approximation up to the order of η^2 inside the exponent of Eq. (3). Since $\hat{S}^z$ does not commute with $\hat{\sigma}_1^{-\Delta\varphi/2}\otimes\hat{\sigma}_2^{\Delta\varphi/2}$, there is a $[\hat{S}^z,\hat{\sigma}_1^{-\Delta\varphi/2}\otimes\hat{\sigma}_2^{\Delta\varphi/2}]$ term $\propto\eta^3$ in the time evolution of Eq. (3). Those terms cannot be compensated by spin-echo pulses. In order to reduce the effect, we can add $\delta_{\rm off}$ to both laser frequencies. The $\delta_{\rm off}$ makes the coupling between $|DD\rangle$and $|SS\rangle$ off-resonant by $2\delta_{\rm off}$, since the same electronic states are connected through red detuned and blue detuned lasers. Only the population transfer between $|DS\rangle$ and $|SD\rangle$ is, therefore, resonant and dominant in MS coupling. By taking only resonant terms, the MS coupling in Eq. (4) leads to $\frac{4}{3}\hbar\eta|\Omega_{\rm eff}|(\hat{\sigma}_1^+\otimes\hat{\sigma}_2^- e^{i\Delta\varphi})+\text{H.c.}$ due to the offset detuning $\delta_{\rm off}$. Since $\hat{\sigma}_1^+\otimes\hat{\sigma}_2^- e^{i\Delta\varphi}$ commutes with $\hat{S}^z$, Eqs. (5) and (6) are valid at η^3 order inside the exponent of Eq. (3). The induced phase error from adding $\delta_{\rm off}$ is compensated by the spin-echo sequence. The black solid curve in Fig. 2(a) shows that including the offset detuning $\delta_{\rm off}$ lowers the error to around 10^{-4} order even at large η.

In order to reduce the spontaneous emission during the gate, fast gate operation is important especially for optical-transition qubits. For the metastable states, the spontaneous emission probability $P_{\rm sp}$ during one gate is determined by the ratio between the operation time and lifetime of the states. The gate time of the proposed gate is similar to the σ^z geometric phase gate with hyperfine states [3], the MS gate [4,10,11], and the Cirac-Zoller gate[12–14] for the same laser intensity, because in all of these cases the coupling strength is that of the first sideband interaction. The maximum intensity is, however, limited by the off-resonant excitations, $\Omega<|\Delta_{1(2)}|\approx\nu/2$. The gate time $2\pi/\delta=2\pi/\frac{4\eta\Omega^2}{\nu}$, therefore, is limited by $2\pi/\eta\nu$. To guarantee that $P_{\rm sp}<10^{-4}$, for Ca^+ ions the operation time has to be faster than 100 μs. This means that it is necessary to increase Ω close to the detunings $|\Delta_{1(2)}|$—as shown at dashed lines in Fig. 2(b)—and also to use large—η as shown in Fig. 2(a).

When Ω is comparable to the detunings, the direct coupling term from off-resonant excitations, $\hat{H}_d=\Sigma_{j=1,2}(\hat{\sigma}_1^++\hat{\sigma}_2^+)\frac{\hbar\Omega_j}{2}e^{-i(\Delta_j t+\phi_j)}+\text{H.c.}$, neglected in Eqs. (2) and (4), needs to be considered. The term mainly induces population exchanges between electronic states. The population error from off-resonant excitations can be described by $1-(\frac{\Omega}{\nu/2})^2\sin^2(\nu t/4)$ around $t=2\pi/\delta$ [10]. The infidelity can be reduced by either precise control of system parameters, such as $\delta = \nu/2N$ with N as an integer, or by pulse shaping. By using pulse shaping, one can start and end the gate operation with a fairly small Ω by adiabatically changing laser intensities [14,15]. As we can see in Fig. 2(b), the infidelity from the direct coupling can be in the order of 10^{-4} up to $\nu/4$ of Ω by using rise and fall times of 10 μs for the pulses.

In the simulation, for a large Ω we observed a reduction in the Raman coupling strength Ω_{eff} proportional to $(\Omega/\nu)^2$. We believe that this reduction is due to an admixture of the other electronic state due to, for instance, off-resonant excitations. The Raman coupling $\Omega_{\text{eff},S}$ of the $|S\rangle$ state has an opposite sign as compared to the coupling $\Omega_{\text{eff},D}$ of the $|D\rangle$ state, $\Omega_{\text{eff},S} = -\Omega_{\text{eff},D}$. Thus the contributions of other electronic states due to off-resonant excitations reduce the strength of the coupling Ω_{eff} proportional to $(\Omega/\nu)^2$.

Furthermore, the amount of the reduction at $\Omega_{\text{eff},S}$ is slightly dependent on η, which might come from the Debye factor [10]. We note that the infidelities of Fig. 2(a) and 2(b) are obtained after correcting the reduction in Ω_{eff}. In experiments, however, the change in Ω_{eff} due to direct coupling is not a problem at all, since the intensities of the bichromatic lasers will be determined experimentally.

We also carefully studied other experimental imperfections, such as the intensity fluctuations of both laser beams, positioning errors of the laser beams on the ions, fluctuations of the laser and the trap frequency, and the occupations of the bus mode and spectators' modes. The proposed gate is quite robust to those imperfections similar to the geometric hyperfine gate of [3]. According to our simulations, for intensity fluctuations of about 10^{-2}, a few tenths of kHz of laser frequency fluctuations, and a few Hz trap frequency fluctuations—less than 0.5 motional quanta of the all motional modes—allow for an infidelity on a level of 10^{-4}.

In conclusion, we propose a σ^z geometric phase gate for optical-transition qubits. The gate has a small spontaneous emission during the operation, and can be applied to magnetic field insensitive states. We analyze and simulate the gate in detail and show that the gate allows one to achieve a high fidelity implementation. The proposed σ^z gates are interesting not only due to the high fidelity, but also to their applicability to decoherence-free subspace constituted by the logical qubits $|DS\rangle, |SD\rangle$ [8,16,17]. If we apply the laser beams described in the paper to two physical qubits that belong to the different logical qubits, the scheme works as the entangling gate for the two logical qubits.

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