Teleportation with atoms

H. Häffner[1,2], M. Riebe[1], F. Schmidt-Kaler[1], W. Hänsel[1], C. Roos[1], M. Chwalla[1], J. Benhelm[1], T. Körber[1], G. Lancaster[1], C. Becher[1] D.F.V. James[3] and R. Blatt[1,2]

[1]Institut für Experimentalphysik, Innsbruck, Austria

[2]Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Austria

[3] Theoretical Division T-4, Los Alamos National Laboratory, Los Alamos NM 87545, United States

Abstract

The teleportation of an atomic state accomplishes the complete transfer of information from one particle to another, employing the non-local properties of quantum mechanics. Recently, two groups have achieved the deterministic teleportation of a quantum state between a pair of trapped ions. Following closely the original proposal of Bennett et *al.*[1], a highly entangled pair of ions is created, a complete Bell-state projective measurement involving the source ion and one of the entangled pair is carried out, and state reconstruction conditioned on this measurement is performed on the other half of the entangled pair.

1 Introduction

Teleportation exploits some of the most fascinating features of quantum mechanics, in particular *entanglement*, shedding new light on the essence of quantum information. It is possible to transfer the quantum information contained in a two-level system –a qubit– by communicating two classical bits and using entanglement. Thus quantum information can be broken down in a purely classical part and a quantum part. Furthermore, teleportation is not merely a simple swapping of quantum states: it does not need a quantum channel to be open at the time the transfer is carried out. Instead it uses the non-local properties of quantum mechanics (entanglement), established by a quantum channel *prior to the generation of the state to be teleported*. Once that link has been established an unknown state can be transferred at any later time using classical communication only. This is quite surprising since the quantum part of the transfer seems to have happenend before the state to be transferred exists. In addition to the motivation to demonstrate and to understand quantum physics, teleportation might have also considerable impact on a future quantum computer as it facilitates the scalability of many quantum computer designs [2].

Teleportation was already demonstrated with photonic qubits [3, 4, 5, 6, 7]. However, these experiments did not include complete two-photon Bell state measurements. In addition, successful teleportation events were established by selecting the data after completion of the experiment, searching for the subset of experiments in which the outcome of the measurement and a preset reconstruction operation were matched: i.e.

teleportation was performed post-selectively. In contrast to this the experiment by Furusawa et *al.* [8] demonstrated unconditional teleportation of continuous variables. Similarly Nielsen et *al.* [9] implemented a deterministic teleportation algorithm with highly mixed states in an liquid-state NMR set-up.

Recently two groups realized quantum teleportation of atomic qubits. The Boulder group [10] teleported the quantum information contained in one Beryllium-ion to another one, while the Innsbruck group [11] used Calcium ions for the same purpose. Both experiments used ions trapped in Paul traps. However, different approaches were pursued: In Boulder the qubits are encoded in the hyperfine structure of the ions, while in Innsbruck the qubit states are stored in superpositions of a ground and metastable electronic state. Furthermore the Boulder group uses segmented traps to perform the required selective read-out of the quantum register, whereas in Innsbruck tightly focused laser beams together with selective excitation of the Zeeman levels are employed for this purpose. Finally the Boulder group chose to work with a geometric phase gate[12], while the Innsbruck group uses composite pulses to realize the phase gate[13]. Despite these different approaches both experiments yield similar results. This demonstrates that ions traps are versatile devices for coherent state manipulation and quantum information processing.

2 Implementation of the Teleportation

The teleportation of a state from a source qubit to a target qubit requires three qubits: the sender's source qubit and an ancillary qubit that is maximally entangled with the receiver's target qubit providing the strong quantum correlation. In our experiments, each qubit is respresented by a superposition of the $S_{1/2}(m_j = -1/2) \equiv |S\rangle$ ground state and the $D_{5/2}(m_j = -1/2) \equiv |D\rangle$ metastable state of a $^{40}\mathrm{Ca}^+$ ion. All three ions are stored together in a linear Paul trap and arrange themselves as a string with an inter-ion distance of 5 μm. Each qubit can be individually manipulated by a series of laser pulses on the $|S\rangle \rightarrow |D\rangle$ quadrupole transition near 729 nm employing narrow-band laser radiation tightly focused onto individual ions in the string. The qubits are initialized in $|S\rangle$ by optical pumping. The ion string's center-of-mass vibrational mode ($\omega = 2\pi \times 1.2$ MHz) is cooled to the ground state as required for controlled interaction between the ions according to the original proposal by Cirac and Zoller[14]. For further experimental details see ref. [15].

The quantum teleportation circuit is displayed in Fig. 1. The circuit is formally equivalent to the one proposed by Bennett et *al.* [1], but adapted to the ion-based quantum processor and can be broken up in the following tasks:

1. **Creation of Bell states**
   We use a pulse sequence of three laser pulses (cf. Tab. 1). Tomography [16, 17] of the created Bell state $(|DS\rangle + |SD\rangle/\sqrt{2}$ shows a fidelity of up to 96% for the entangling operation. In addition, our experiment show that the lifetimes of Bell states of the type $(|DS\rangle + e^{i\phi}|SD\rangle/\sqrt{2}$ approach the fundamental limit given by the spontaneous decay rate of the metastable $\mathrm{D}_{5/2}$-level of 1.2 s. After quantum link between the source and the target resions is established, we prepare a test state χ via a single qubit operation U_χ on the source ion.

2. **Rotation into the Bell-basis**
   A Bell state measurement can be accomplished by rotating the basis of the source and the ancilla ion into the Bell basis before the actual read-out of the

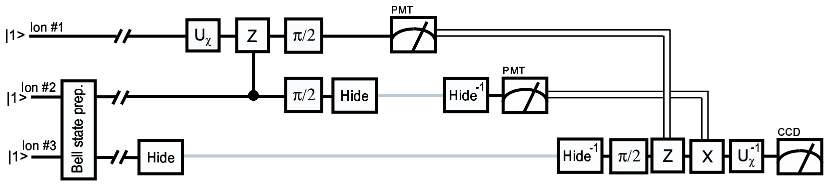

Figure 1: The teleportation algorithm's quantum circuit. Double lines represent flow of classical information, whereas single lines flow of quantum information. The shaded lines indicate when a qubit is protected from detection light via so-called hiding-pulses. First ion #2 and #3 are entangled creating the quantum link between the source region (ions #1 and #2) and the target ion (ion #3). Then after some waiting time the state to be teleported is prepared via the unitary operation U_χ. A joint Bell state measurement realized with the controlled Z-gate together with detection via a photomultiplier tube (PMT) realizes the Bell state measurement. After the reconstruction pulses the success of the teleportation is tested by appling the inverse preparation procedure before measuring the target ion on an intensified CCD-camera (Charged Coupled Device).

qubits. This rotation is implemented with a controlled-Z (phase) gate and appropriate single qubit operations. The experimental implementation of the controlled-Z-gate is described in ref. [13]. To illustrate the rotation into the Bell-basis more easily, we will use in the following a zero-controlled-not (0-CNOT) gate as a substitute for the controlled Z-gate: suppose one has the Bell state $(|DS\rangle + |SD\rangle)/\sqrt{2}$ (note that we use the convention $|D\rangle \equiv |0\rangle$ and $|S\rangle \equiv |1\rangle$), then application of a 0-CNOT followed by a $\pi/2$-Carrier-Pulse on the control bit (the leftmost bit) yields:

$$(|DS\rangle+|SD\rangle)/\sqrt{2} \stackrel{0-CNOT}{\longrightarrow} (|DD\rangle+|SD\rangle)/\sqrt{2} = (|D\rangle+|S\rangle)|D\rangle/\sqrt{2} \stackrel{R_C^{\mathrm{C}}(\pi/2,0)}{\longrightarrow} |SD\rangle \quad (1)$$

The pulse $R_C^{\mathrm{C}}(\pi/2, 0)$ denotes a single qubit rotation of length $\pi/2$ with phase 0. Now we have mapped the Bell state $|DS\rangle + |SD\rangle$ to $|SD\rangle$. Similarly all other Bell states are mapped onto orthogonal logical eigenstates. Therefore a measurement in the logical eigenbasis yields now a precise knowledge of the original Bell state.

3. **Selective read-out of the quantum register and conditional quantum gates**
The measurement process must preserve the coherence of the target qubit, ion #3. Thus, the state of ion #3 is hidden by transferring it to a superposition of levels which are not affected by the detection light. We employ an additional Zeeman level of the $D_{5/2}$ manifold for this purpose. Applying now laser light at 397 nm for 250 μs to the ion crystal, only the ion in question can fluoresce, and that only if it is the $S_{1/2}$-state [18]. This hiding technique is also used to sequentially read out ion #1 and ion #2 with a photomultiplier tube (see Fig. 2). Instead of using a CCD-camera (which can easily distinguish be-

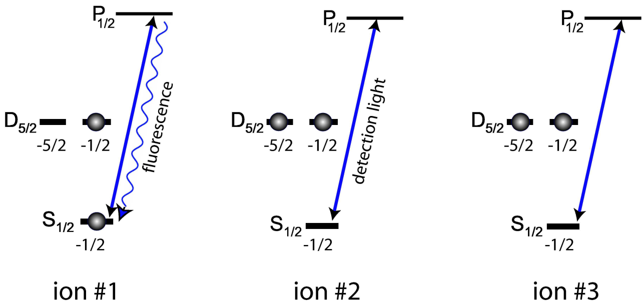

Figure 2: Partial level scheme of the three Ca-ions. Only ion #1 is read out. Ion #2 and #3's quantum information is protected in the Zeeman manifold of the $D_{5/2}$-level, namely the $m_J = -1/2$ and $m_J = -5/2$ levels.

tween different ions), we prefer to take advantage of the fast electronic read-out capabilities of a photo-multiplier tube. This ensures a reaction on the measurement result within the single qubit coherence time. A digital electronic circuit counts the number of detected photons and compares it to the threshold (less than 6 detected photons indicate that the ion is in the $D_{5/2}$ level).

Conditioned on the measurement result, we apply single qubit rotations on the target ion [18]. This is implemented by using a classical AND-gate between the output of the electronic circuit which has stored the measurement result and the output of a Digital board on which the reconstruction pulses are programmed. Thus, we apply the appropriate unitary qubit rotation, $-i\sigma_y$, $-i\sigma_z$, $i\sigma_x$, or 1 (with Pauli operators σ_k) to reconstruct the state in the target ion #3, obtaining χ on ion #3. Note that to realize $-i\sigma_y$, $\sigma_x\sigma_z$ is implemented. This has the advantage that we can apply σ_x if ion #1 is measured to be in $|D\rangle$ and σ_z if ion #2 is measured to be in $|D\rangle$ and can keep so the electronic logic quite simple.

The whole pulse sequence is displayed in Tab. 1. In contrast to Fig.1, here also spin echo pulses are included. The conditioned pulses #31,32,33 are applied only if less than 6 photon detection events were recorded during the respective detection time of 250 μs. The phase ϕ for the pulses is fixed during all experiments. It is used to compensate for the 50 Hz related magnetic field fluctuations during the execution of the teleportation algorithm.

To obtain directly the fidelity of the teleportation, we perform on ion #3 the inverse of the unitary operation U_χ^{-1} used to create the input state $|\chi\rangle$ from state $|S\rangle$ (see pulses #9 and #34 in Tab. 1). The teleportation is successful if and only if ion #3 is always found in $|S\rangle$. The teleportation fidelity, given by the overlap $F = \langle S|U_\chi^{-1}\rho_{\exp}U_\chi|S\rangle$, is plotted in Fig. 3 for all four test states $\{|S\rangle, |D\rangle, |S+D\rangle, |S+iD\rangle\}$.

3 Results

The obtained fidelities range from 73% to 76%. Teleportation based on a completely classical resource instead of a quantum entangled resource yields a maximal possible

		Action	Comment
	1	Light at 397 nm	Doppler preparation
	2	Light at 729 nm	Sideband cooling
	3	Light at 397 nm	Optical pumping
Entangle	4	$R_3^+(\pi/2, 3\pi/2)$	Entangle ion #3 with motional qubit
	5	$R_2^{\mathrm{C}}(\pi, 3\pi/2)$	Prepare ion #2 for entanglement
	6	$R_2^+(\pi, \pi/2)$	Entangle ion #2 with ion #3
	7	Wait for 1μs – 10 000 μs	Stand-by for teleportation
	8	$R_3^{\mathrm{H}}(\pi, 0)$	Hide target ion
	9	$R_1^{\mathrm{C}}(\vartheta_\chi, \varphi_\chi)$	Prepare source ion #1 in state χ
Rotate into Bell-basis	10	$R_2^+(\pi, 3\pi/2)$	Get motional qubit from ion #2
	11	$R_1^+(\pi/\sqrt{2}, \pi/2)$	Composite pulse for phasegate
	12	$R_1^+(\pi, 0)$	Composite pulse for phasegate
	13	$R_1^+(\pi/\sqrt{2}, \pi/2)$	Composite pulse for phasegate
	14	$R_1^+(\pi, 0)$	Composite pulse for phasegate
	15	$R_1^{\mathrm{C}}(\pi, \pi/2)$	Spin echo on ion #1
	16	$R_3^{\mathrm{H}}(\pi, \pi)$	Unhide ion #3 for spin echo
	17	$R_3^{\mathrm{C}}(\pi, \pi/2)$	Spin echo on ion #3
	18	$R_3^{\mathrm{H}}(\pi, 0)$	Hide ion #3 again
	19	$R_2^+(\pi, \pi/2)$	Write motional qubit back to ion #2
	20	$R_1^{\mathrm{C}}(\pi/2, 3\pi/2)$	Part of rotation into Bell-basis
	21	$R_2^{\mathrm{C}}(\pi/2, \pi/2)$	Finalize rotation into Bell basis
Read-out	22	$R_2^{\mathrm{H}}(\pi, 0)$	Hide ion #2
	23	PMDetection for 250 μs	Read out ion #1 with photomultiplier
	24	$R_1^{\mathrm{H}}(\pi, 0)$	Hide ion #1
	25	$R_2^{\mathrm{H}}(\pi, \pi)$	Unhide ion #2
	26	PMDetection for 250 μs	Read out ion #2 with photomultiplier
	27	$R_2^{\mathrm{H}}(\pi, 0)$	Hide ion #2
	28	Wait 300 μs	Let system rephase; part of spin echo
	29	$R_3^{\mathrm{H}}(\pi, \pi)$	Unhide ion #3
	30	$R_3^{\mathrm{C}}(\pi/2, 3\pi/2 + \phi)$	Change basis
Recon-struction	31	$R_3^{\mathrm{C}}(\pi, \phi)$	iσ_x } $= -i\sigma_z$ conditioned on PMDetection #1
	32	$R_3^{\mathrm{C}}(\pi, \pi/2 + \phi)$	-iσ_y }
	33	$R_3^{\mathrm{C}}(\pi, \phi)$	iσ_x conditioned on PMDetection #2
	34	$R_3^{\mathrm{C}}(\vartheta_\chi, \varphi_\chi + \pi + \phi)$	Inverse of preparation of χ with offset ϕ
	35	Light at 397 nm	Read out of ion #3 with camera

Table 1: To implement the teleportation, we use pulses on carrier transitions $R_i^C(\theta, \varphi)$ and $R_i^H(\theta, \varphi)$ (no change of the motional state of the ion crystal) and, additionally, on the blue sideband $R_i^+(\theta, \varphi)$ (change of the motional state) on ion i. The index C denotes carrier transitions between the two logical eigenstates, while the index H labels transitions from the $S_{1/2}(m_J = -1/2)$ to the $D_{5/2}(m_J = -5/2)$-level. For the definitions of $R_i^{C,H,+}(\theta, \varphi)$ see the refs. [19] and [13].

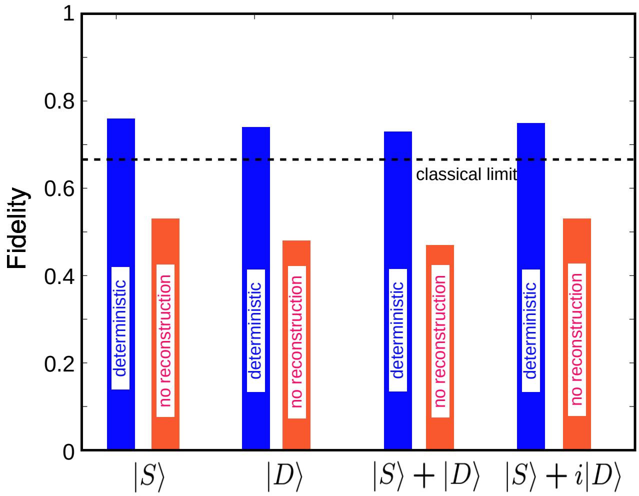

Figure 3: Result of the teleportation: The four test states are teleported with fidelities of 76%, 74%, 73%, and 75%, respectively. For each input state 300 single teleportation experiments were performed. The error of each entry, estimated from quantum projection noise, is 2.5%. We also show the results if the reconstruction operations are omitted, yielding an average fidelity of 49.6%. The optimum teleportation obtainable by purely classical means and no assumptions about the initial states reaches a fidelity of 66.7% (dashed line).

fidelity of 66.7% [21] (dashed line in Fig. 3). Note that this classical boundary holds only if no assumptions on the states to be teleported are made. If one restricts oneself to only the four test states, strategies exist which use no entanglement and yield fidelities of 78% [20]. However, each of these strategies must be designed for a specific test state ensemble to work properly. Note also, that to rule out hidden variable theories, a fidelity in excess of 0.87 is required [22].

For comparison, we also show data where the reconstruction pulses were not applied. Without the classical information communicated from the Bell measurement, the receiver's state is maximally mixed, i.e. there is no information available on the source state. Also, the measurement outcomes of ions #1 and #2 do not contain any information about the initial state. Indeed we find each possible result with an equal probability of 0.25 ± 0.036, independent of the test input states. Note, that only with both the receiver's qubit and the result of the Bell measurement, the initial state can be retrieved.

We emphasize that the conditional, deterministic reconstruction step, in combination with the complete Bell state analysis, is one of the crucial improvements with respect to former experimental realizations of quantum teleportation. Furthermore, after the teleportation procedure the state χ is always available and may be used for further experiments.

To emphasize the role of the shared entangled pair as a resource, we store the Bell state for some time and then use it only later (after up to 20 ms) for teleportation. For waiting times of up to 20 ms (exceeding the time we require for the teleportation by a factor of 10) we observe no decrease in the fidelity. For longer waiting times, we expect the measured heating of the ion crystal of smaller than 1 phonon/100 ms to reduce the fidelity significantly. This is because for a successful rotation into the Bell-basis we require the bus mode of the ion crystal to be in the ground state.

4 Experimental Imperfections

The obtainable fidelity is limited mainly by dephasing mechanisms. The most obvious one are frequency fluctuations of the laser driving the qubit transition and magnetic field fluctuations. Since these fluctuations are slow as compared to the execution time of 2 ms, they can be cancelled to some degree with spin echo techniques [23]. However, during the algorithm we have to use different pairs of states to encode the quantum information, one of the pairs being only sensitive to magnetic field fluctuations the other one being sensitive to both laser and magnetic field fluctuations. To overcome these complications two spin echo pulses are introduced (see Tab. 1). Its optimal position in time was determined with numerical simulations. From measurements we estimate that the remaining high frequency noise reduces the fidelity by about 5%. Another source of fidelity loss is an imperfect AC-Stark shift compensation. AC-Stark compensation is needed to get rid of the phase shifts introduced by the laser driving the weak sideband transition due to the presence of the strong carrier transitions[24]. Recent measurements suggest that an imperfect compensation as introduced by the incorrect determination of the sideband frequency by only 100 Hz lead to a loss of teleportation fidelity in the order of 5 %.

Imperfect state detection as introduced by a sub-optimal choice for the threshold (6 instead of 3 counts) was analyzed later to contribute in the order of 3 %. However, the biggest contribution to the read-out error stems from an incorrect determination of of the hiding pulses' frequency and Rabi-frequency and amounts to 7%.

Addressing errors on the order of 3–4% were estimated via numerical simulations to reduce the fidelity by about 6 %. The addressing errors were measured by comparing the Rabi flopping frequency between neighboring ions and corresponds to a ratio of 10^{-3} in intensity between the addressed ion and the other ones.

Treating these estimated error sources independently yields an expected fidelity of 77 % in good agreement with the experimental findings.

In conclusion, we described an experiment demonstrating teleportation of atomic states. The experimental procedures might be applied in future quantum information processing networks: with long lived entangled states as a resource, quantum teleportation can be used for the distribution of quantum information between different nodes of the network.

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